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On symplectic linearizable actions

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Abstract: We prove that linearizable actions are also symplectically linearizable (either smoothly or analytically) in a neighborhood of a fixed point. Specifically, the fundamental vector fields associated with the action can be simultaneously linearized in Darboux coordinates. This result extends equivariant symplectic local normal forms to non-compact group actions.

In both formal and analytic frameworks, the existence of linearizing coordinates is tied to a cohomological equation, which admits a solution for semisimple actions [9, 8]. Consequently, an analytic symplectic action of a semisimple Lie algebra can be locally linearized in Darboux coordinates, enabling the simultaneous analytic linearization of Hamiltonian vector fields near a shared zero. However, in the smooth setting, this result is restricted to semisimple Lie algebras of compact type. We construct an explicit example of a smooth, non-linearizable Hamiltonian action with a semisimple linear part, thereby answering in the negative a question posed by Eliasson [5].

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1 Introduction

A classical result due to Bochner [1] establishes that a compact Lie group action on a smooth manifold is locally equivalent, in the neighbourhood of a fixed point, to its linearization. This result holds in the C^k category. It is worth exploring if similar results hold in the non-compact case.

As observed in [8], if the Lie group is connected, the linearization problem can be formulated in the following terms: find a linear system of coordinates for the vector fields corresponding to the one-parameter subgroups of G; or more generally, consider the representation of a Lie algebra and find coordinates on the manifold that simultaneously linearize the vector fields in the image of the representation vanishing at a point. This is the perspective we adopt in this note when referring to *linearization*.

In the formal and analytic cases, the existence of coordinates that linearize the action is related to a cohomological equation that can always be solved when the Lie group under consideration is semisimple [9], [8]. Guillemin and Sternberg also studied the problem in the C^∞ setting. At the end of [8], they presented the celebrated example of a non-linearizable action of $\mathfrak{sl}(2,\mathbb{R})$ on $\mathbb{R}^3$, constructed via a perturbation involving the radial vector field with flat coefficients. This example has been foundational in the literature, inspiring the construction of other examples with profound geometric implications, such as Weinstein's non-stable Poisson structure example [19].

When the semisimple Lie algebras are of compact type, the linearization of the action can be achieved by combining the local integration of the Lie group action with Bochner's theorem, leading to the linearization of the associated Lie algebra action [6].

Linearization techniques also play a significant role in Hamiltonian systems. When a Hamiltonian system arises from a symplectic action of a compact Lie group fixing a point, the equivariant version of Darboux's theorem ([18], [3]) ensures that the group action can be linearized in Darboux coordinates near the fixed point. It is worth exploring if similar results apply beyond the compact case.

For complete integrable systems, an associated abelian symplectic action emerges.

When the integrable system in local coordinates has a “linear part" linked to a Cartan subalgebra, this leads to non-degenerate singularities [4]. As shown in [5], [4], [10], [11] and [12], complete integrable systems near non-degenerate singular points are equivalent to their linear models. Consequently, the Hamiltonian system itself is equivalent to the linear one. This result provides normal forms for integrable systems near singular non-degenerate points and, specifically, ensures the simultaneous linearization of Hamiltonian vector fields near a common zero.

The next challenge involves Hamiltonian systems with a semisimple linear part, as proposed by Eliasson in [5]. In the formal or analytic setting, results by Guillemin and Sternberg [8] and Kushnirenko [9] demonstrate that such systems are equivalent to the linear model when the symplectic form is disregarded. In this note, we establish that not only can the Hamiltonian vector fields be linearized, but they can also be linearized in Darboux coordinates.

Following Guillemin and Sternberg’s approach, we prove that if a symplectic Lie algebra action of semisimple type fixes a point, there exist analytic Darboux coordinates in which the analytic vector fields generating the Lie algebra action are linear. This result also extends to complex analytic Lie algebra actions on complex analytic manifolds. Additionally, we construct an example of a Hamiltonian system with a semisimple linear part that is not C^∞-linearizable.

Organization of this article: In Section 2, we prove that linearizable actions on symplectic manifolds can be locally linearized in Darboux coordinates. In Section 3, we apply this to show that any real analytic symplectic action of a semisimple Lie algebra can be linearized in real analytic Darboux coordinates in a neighborhood of a fixed point. Furthermore, this result extends to analytic complex manifolds and complex analytic actions of semisimple Lie algebras. In Section 4, we present a counterexample proving that the linearization result does not hold in general for smooth Hamiltonian actions of semisimple Lie algebras.

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2 Linearizable actions in Darboux coordinates

Let $\mathfrak{g}$ be a *Lie algebra* and let $\rho : \mathfrak{g} \longrightarrow L_{analytic}$ stand for a representation of $\mathfrak{g}$ in the algebra of real (or complex) analytic vector fields on a real (or complex) analytic manifold M.

We say that $p \in M$ is a **fixed point** for ρ if the vector fields in $\rho(\mathfrak{g})$ vanish at p. We say that ρ can be **linearized** in a neighborhood of a fixed point if there exist local coordinates in a neighbourhood of p such that the vector fields in the image of ρ can be simultaneously linearized (i.e, ρ is **equivalent to a linear representation**).

Assume that the Lie algebra action is (analytically/smoothly) linearizable and assume that M is endowed with a symplectic structure (smooth, analytic). We first prove that it is then symplectically linearizable.

Theorem 2.1. *Let $\mathfrak{g}$ be a Lie algebra and let (M, ω) be a (real or complex) analytic symplectic manifold. Let ρ be a representation by analytic symplectic vector fields. Let p be a fixed point for ρ and assume that ρ can be linearized. Then there exist local analytic coordinates $(x_1, y_1, \dots, x_n, y_n)$ in a neighborhood of p for ρ such that ρ is a linear representation and ω can be written as,*

$$\omega = \sum_{i=1}^{n} dx_i \wedge dy_i.$$

Proof. Let ρ be an analytic symplectic action of a Lie algebra on a manifold M, with a fixed point $p \in M$. Choose analytic coordinates $(x_1, y_1, \dots, x_n, y_n)$ centered at p in which the action ρ is linear. Let ω_1 denote the symplectic form in these coordinates. Although ρ is now linear, ω_1 need not be of Darboux type.

We denote by ω_0 the constant (degree-zero) term in the Taylor expansion of ω_1 at the origin. Since ω_1 is preserved by ρ and ρ is linear, it follows that ω_0 is preserved by the linearized action $\rho^{(1)} = \rho$. In particular, ω_0 is a constant symplectic form invariant under the action. Our goal is to construct a local analytic diffeomorphism ϕ, fixing the origin, such that $\phi^*(\omega_1) = \omega_0$ and ϕ commutes with ρ. That is, we seek an equivariant analytic Darboux theorem for ω_1, linearizing the form while preserving the linear action ρ.

To this end, we apply the path method [16] for analytic symplectic structures. By the Poincaré lemma, there exists an analytic 1-form α such that

$$\omega_1 = \omega_0 + d\alpha.$$

Define a path of symplectic forms:

$$\omega_t = t\omega_1 + (1-t)\omega_0, \quad t \in [0,1].$$

Each ω_t is an analytic symplectic form in a neighbourhood of the origin. Moreover, the action ρ preserves both ω_0 and ω_1, hence it preserves the entire path ω_t.

We now define the time-dependent analytic vector field X_t by Moser's equation:

$$i_{X_t}\omega_t = -\alpha. \tag{2.1}$$

In order to ensure that X_t is invariant under ρ, it suffices to construct α invariant under ρ. For this purpose, we apply the standard homotopy operator used in the proof of the Poincaré lemma, adapted to our equivariant setting.

Let $R = \sum x_i\partial_{x_i} + y_i\partial_{y_i}$ be the radial vector field, and h_t the homothety $x \longmapsto tx$. Then, we define

$$\alpha := \int_0^1 \frac{1}{t} h_t^*(i_R\beta)\, dt, \quad \text{where } \beta = \omega_1 - \omega_0.$$

Because β is ρ-invariant and ρ commutes with R, it follows that α is also ρ-invariant. Thus, the vector field X_t is invariant under ρ.

Let ϕ_t denote the flow of X_t, satisfying the differential equation

$$\frac{\partial \phi_t}{\partial t}(q) = X_t(\phi_t(q)), \quad \phi_0 = \text{id}. \tag{2.2}$$

Since X_t is ρ-invariant, the flow ϕ_t commutes with the action ρ. Moreover, because α vanishes at the origin, so does X_t, ensuring that each ϕ_t fixes the origin.

By construction, $\phi_t^*(\omega_t) = \omega_0$, and in particular $\phi_1^*(\omega_1) = \omega_0$. The diffeomorphism $\phi := \phi_1$ is then the desired equivariant analytic transformation taking ω_1 to ω_0 while preserving the linear action ρ.

This completes the proof. $\square$

Remark 2.2. The theorem above is stated in the analytic category; however, if the linearization is assumed to hold in the smooth category, the symplectic diffeomorphism obtained from the proof is also smooth.

3 The case of analytic semisimple Lie algebra actions

Guillemin and Sternberg provided in [8] a complete characterization of analytically linearizable actions. They demonstrated that a necessary and sufficient condition for the representation

$$\rho : g \to L_{\text{analytic}}$$

to be locally analytically linearizable is the existence of an analytic vector field X, defined in a neighborhood of p, vanishing at p, with the identity matrix as its Jacobian at p, and commuting with all the vector fields in $\tilde{g}$.

This condition was elegantly recast in cohomological terms in [8]. They proved that the first cohomology group $H^1(g, V^*)$ acts as an obstruction to analytic linearization. For semisimple g, $H^1(g, V^*)$ vanishes for all representation spaces V, ensuring the possibility of analytic linearization. On the other hand, for non-semisimple g, one can construct a representation space V such that $H^1(g, V^*) \neq 0$, which precludes analytic linearization.

This result establishes the semisimple case as a natural candidate for analytic linearization.

Guillemin and Sternberg [8] and Kushnirenko [9] proved the following.

Theorem 3.1 (Guillemin-Sternberg, Kushnirenko). *The representation $\rho : \mathfrak{g} \longrightarrow L_{analytic}$ with $\mathfrak{g}$ semisimple is locally equivalent, via an analytic diffeomorphism, to a linear representation of $\mathfrak{g}$ in a neighbourhood of a fixed point for ρ.*

As an application of theorem 2.1: When the representation is done by Hamiltonian vector fields (locally symplectic), the analytic diffeomorphism that gives the equivalence of the initial representation to the linear representation can be chosen to take the initial symplectic form to the Darboux one. Namely,

Corollary 3.2. *Let $\mathfrak{g}$ be a semisimple Lie algebra and let (M,ω) be a (real or complex) analytic symplectic manifold. Let $\rho : \mathfrak{g} \longrightarrow L_{analytic}$ be a representation by analytic symplectic vector fields. Then there exist local analytic coordinates $(x_1, y_1, \ldots, x_n, y_n)$ in a neighbourhood of a fixed point p for ρ such that ρ is a linear representation and ω can be written as,*

$$\omega = \sum_{i=1}^{n} dx_i \wedge dy_i.$$

4 Non-linearizable semisimple smooth actions

4.1 The counterexample of Cairns and Ghys

In this section we recall the results of Cairns and Ghys concerning a C^∞-action of $SL(2,\mathbb{R})$ on $\mathbb{R}^3$ which is not linearizable. All results mentioned in this section are contained in section 8 of [2].

Consider the basis $\{X, Y, Z\}$ of $\mathfrak{sl}(2,\mathbb{R})$ satisfying the relations:

$$[X,Y] = -Z, \quad [Z,X] = Y, \quad [Z,Y] = -X$$

Now consider the representation on $\mathbb{R}^3$ defined on this basis as:

$$\begin{aligned} \rho(X) &= y\frac{\partial}{\partial z} + z\frac{\partial}{\partial y} \\ \rho(Y) &= x\frac{\partial}{\partial z} + z\frac{\partial}{\partial x} \\ \rho(Z) &= x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x} \end{aligned} \tag{4.1}$$

The orbits of this action are the level sets of the quadratic form $Q = x^2 + y^2 - z^2 = r^2 - z^2$ (where $r^2 = x^2 + y^2$). These level sets are non-degenerate quadrics: one-sheeted hyperboloids for $Q > 0$, two-sheeted hyperboloids for $Q < 0$, and a quadratic cone for $Q = 0$.

Introduce the radial vector field

$$R = x\,\frac{\partial}{\partial x} + y\,\frac{\partial}{\partial y} + z\,\frac{\partial}{\partial z},$$

and perturb ρ by setting

$$\begin{aligned} \widetilde{X} &= \rho(X) + f\,R\,, \\ \widetilde{Y} &= \rho(Y) + g\,R\,, \\ \widetilde{Z} &= \rho(Z)\,, \end{aligned} \tag{4.2}$$

where

$$f(x,y,z) = x\,A\big(z, \sqrt{x^2+y^2}\big), \quad g(x,y,z) = -\,y\,A\big(z, \sqrt{x^2+y^2}\big),$$

and

$$A(z,r) \;=\; \frac{a\big(r^2 - z^2\big)}{r^2},$$

with $a:\,\mathbb{R}\to\mathbb{R}$ any C^∞–function which vanishes for $r^2 - z^2 \le 0$ and is bounded.

By [2], the fields $\widetilde{X}, \widetilde{Y}, \widetilde{Z}$ still close under the Lie bracket to an $\mathfrak{sl}(2,\mathbb{R})$–algebra and are complete. Hence they integrate to an action $\hat{\rho}$ of the universal cover of $SL(2,\mathbb{R})$, which descends to $SL(2,\mathbb{R})$ itself since $\widetilde{Z} = \rho(Z)$ is unchanged. Moreover:

- On the “hyperbolic region” $\{x^2+y^2 > z^2\}$, one has $a(r^2 - z^2) \neq 0$ and $\{\widetilde{X}, \widetilde{Y}, \widetilde{Z}\}$ are linearly independent, so $\hat{\rho}$–orbits are 3-dimensional.
- On and inside the “cone” $\{x^2+y^2 \le z^2\}$, one has $a(r^2 - z^2) = 0$ so $\widetilde{X} = \rho(X)$, $\widetilde{Y} = \rho(Y)$, and $\hat{\rho}$ coincides with the linear action.

Since the original linear action never has 3-dimensional orbits, $\hat{\rho}$ cannot be conjugate to it. Therefore, the deformed action is not linearizable.

4.2 The counterexample of Guillemin and Sternberg

The construction of Guillemin and Sternberg [8] follows the guidelines outlined below. It is quite similar to the counterexample of Grant and Cairns; however, the key difference is that the vector field the perturbation does not preserve Z, so it cannot be guaranteed that it lifts to $SL(2,\mathbb{R})$.

If we perturb the initial action of $\mathfrak{sl}(2,\mathbb{R})$ to the non-linear action:

$$\hat{\rho}(X) = \rho(X) + \frac{xz}{r^2} g(r^2 - z^2)R,$$
$$\hat{\rho}(Y) = \rho(Y) - \frac{yz}{r^2} g(r^2 - z^2)R,$$
$$\hat{\rho}(Z) = \rho(Z) + g(r^2 - z^2)R,$$

where $R = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}$ is the radial vector field, and $g \in C^\infty(\mathbb{R})$ is such that $g(x) > 0$ if $x > 0$, and $g(x) = 0$ if $x \leq 0$.

Inside the cone $r^2 - z^2$, the two sets of vector fields are identical. However, if we choose $g(u) = e^{-1/u^2}$, $u > 0$, and $g(u) = 0$, $u \leq 0$, for example, then outside the cone $r^2 - z^2 = 0$, the vector field $\rho(X) = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}$ has closed circular orbits, while the corresponding deformed vector field $\hat{\rho}(X)$ has orbits that spiral outward.

Therefore, it is impossible to find a C^∞-mapping defined in a neighborhood of the origin. Hence, ρ is not linearizable.

4.3 A Hamiltonian counterexample

We construct a counterexample to C^∞–linearization under the hypothesis that the action is Hamiltonian, thereby giving a negative answer to a question of Eliasson [5]. We keep the notation of the Cairns–Ghys construction from Section 4.1.

Proposition 4.1. *Let α be the $SL(2,\mathbb{R})$–action on $\mathbb{R}^3$ generated by*

$$\overline{X} = \rho(X) + fR, \qquad \overline{Y} = \rho(Y) + gR, \qquad \overline{Z} = \rho(Z),$$

where $R = x\,\partial_x + y\,\partial_y + z\,\partial_z$ and

$$f = x\,A\Big(z, \sqrt{x^2+y^2}\Big), \qquad g = -y\,A\Big(z, \sqrt{x^2+y^2}\Big), \qquad A(z,r) = \frac{a(r^2 - z^2)}{r^2},$$

with $a : \mathbb{R} \to \mathbb{R}$ smooth, bounded, and vanishing on $\mathbb{R}^-$. Let $\widehat{\alpha}$ be the cotangent lift of α to $T^*(\mathbb{R}^3)$. Then $\widehat{\alpha}$ is Hamiltonian and not C^∞–linearizable in a neighbourhood of the origin.

Proof. For a diffeomorphism $g : M \to M$, the cotangent lift is

$$\widehat{g}(q,p) = \big(g(q),\, (dg_q^{-1})^* p\big),$$

so

$$\pi\circ\widehat{g} = g\circ\pi \qquad \text{and} \qquad \widehat{g}(q,0) = (g(q),0). \tag{4.3}$$

Thus fibres map to fibres and the zero section is preserved by *every* cotangent lift. In particular, with

$$F := \pi^{-1}(0) = \{x = y = z = 0\}, \qquad Z := \{a = b = c = 0\}, \qquad O := (0,0,0;0,0,0),$$

both F and Z are invariant for the lifted linear action and for the lifted Cairns–Ghys action (note that 0 is fixed in the base, since ρ is linear and fR, gR vanish at 0).

Let $\alpha^{(1)}$ denote the linear part of α and $\widehat{\alpha}^{(1)}$ its cotangent lift. The latter is Hamiltonian for $\omega = d\theta$ ($\theta = a\,dx + b\,dy + c\,dz$), with moment map

$$\mu = (zb + cy,\ az + cx,\ -ay + bx) \in \mathfrak{sl}(2,\mathbb{R})^*,$$

whose Jacobian is

$$D\mu = \begin{pmatrix} 0 & c & b & 0 & z & y \\ c & 0 & a & z & 0 & x \\ b & -a & 0 & -y & x & 0 \end{pmatrix}.$$

For Hamiltonian actions one has $\dim(G\cdot m) = \operatorname{rank} d\mu_m$ (see, for instance, [7, §26]). A direct computation shows that

$$\operatorname{rank} D\mu = \begin{cases} 0, & p = O, \\ 2, & (x,y,z)\times(-a,-b,c) = 0,\ p \neq O, \\ 3, & \text{otherwise.} \end{cases}$$

Equivalently, the rank drops to 2 precisely when (x,y,z) is collinear with $(-a,-b,c)$, a locus containing $F = \{x = y = z = 0\}$ and $Z = \{a = b = c = 0\}$ but strictly larger. Consequently,

$$\dim \mathcal{O}_{\widehat{\alpha}^{(1)}}(p) = \begin{cases} 0, & p = O, \\ 2, & (x, y, z) \times (-a, -b, c) = 0,\ p \neq O, \\ 3, & \text{otherwise}, \end{cases} \tag{4.4}$$

and, in particular, $\dim \mathcal{O}_{\widehat{\alpha}^{(1)}}(q, 0) = 2$ for all $q \neq 0$.

For any lifted action, the fundamental vector fields satisfy

$$\widehat{\xi}_X(q, p) = \big(\xi_X(q),\ -p\circ d\xi_X(q)\big). \tag{4.5}$$

Hence along Z one has $\widehat{\xi}_X(q, 0) = (\xi_X(q), 0)$, so every orbit starting in Z stays in Z. Conversely, no orbit through a point $(q, p) \notin Z$ can be contained in Z, since it already contains $(q, p) \notin Z$. Thus, near O, the orbits contained in Z are exactly those starting in Z. Moreover, from (4.5) we read off

$$\dim \mathcal{O}^{\widehat{\alpha}}(q, 0) = \dim \mathcal{O}^{\alpha}(q).$$

By [2, §8] there exist points $q \to 0$ in the base with 3–dimensional α–orbits. For such q, put $p = (q, 0) \in Z \setminus \{O\}$. Then

$$\dim \mathcal{O}^{\widehat{\alpha}}(q, 0) = \dim \mathcal{O}^{\alpha}(q) = 3. \tag{4.6}$$

Assume, for contradiction, that there exists a germ $\Phi : (T^*M, O) \to (T^*M, O)$ with $\Phi\circ\widehat{\alpha}^{(1)} = \widehat{\alpha}\circ\Phi$. Conjugacy carries orbits diffeomorphically to orbits, preserving their dimension. Since, near O, the orbits contained in Z are precisely those starting in Z for both lifted actions, necessarily $\Phi(Z) = Z$ and $\Phi(F) = F$. Hence $\Phi^{-1}(p) \in Z \setminus \{O\}$, so by (4.4)

$$\dim \mathcal{O}^{\widehat{\alpha}^{(1)}}\big(\Phi^{-1}(p)\big) = 2,$$

whereas by (4.6) $\dim \mathcal{O}^{\widehat{\alpha}}(p) = 3$, a contradiction. Therefore, $\widehat{\alpha}$ is not C^∞–linearizable near the zero section. □

Remark 4.2. We can employ the same strategy, adopting the counterexample by Guillemin and Sternberg in the process.

Consider the Lie algebra action of $\mathfrak{sl}(2,\mathbb{R})$, denoted by $\overline{\rho}$, on $\mathbb{R}^3$, generated by the vector fields:

$$\hat{\rho}(X) = \rho(X) + \frac{xz}{r^2}g(r^2 - z^2)R,$$
$$\hat{\rho}(Y) = \rho(Y) - \frac{yz}{r^2}g(r^2 - z^2)R,$$
$$\hat{\rho}(Z) = \rho(Z) + g(r^2 - z^2)R,$$

where $R = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}$ is the radial vector field, and $g \in C^\infty(\mathbb{R})$ satisfies $g(u) = e^{-1/u^2}$ for $u > 0$ and $g(u) = 0$ for $u \leq 0$.

Using similar guidelines to those of Guillemin and Sternberg in [8], we can verify that the lifted action to $T^*(\mathbb{R}^3)$ is not C^∞-linearizable.

The lift of the action can be computed using the Liouville one-form. Let $\theta = a\,dx + b\,dy + c\,dz$. Then, the lift of the non-perturbed vector field is a Hamiltonian vector field with the Hamiltonian function

$$f = -ay + bx,$$

and the lifted vector field of the perturbed system is the Hamiltonian vector field with respect to the function

$$f' = -ay + bx + g(r^2 - z^2)(ax + by).$$

The Hamiltonian vector field of f is given by:

$$x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x} + a\frac{\partial}{\partial b} - b\frac{\partial}{\partial a},$$

and it exhibits periodic orbits. In contrast, the corresponding deformed vector field, the Hamiltonian vector field of f', has orbits that spiral outward.

4.4 The case of semisimple Lie algebras of compact type

When the Lie algebra action is of compact type, it can be integrated into an action of a compact Lie group G (see [6] for a proof, which is based on the use of algebroids).

Given a fixed point for the action p, we can associate a linear action of the group in a neighbourhood of p, with the group action preserving the symplectic structure (which we can assume to be in Darboux coordinates). Applying the equivariant Darboux theorem [3], we obtain a diffeomorphism ϕ that linearizes the group action G in Darboux coordinates. By differentiation, this provides the linearization of the Lie algebra action ρ.

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